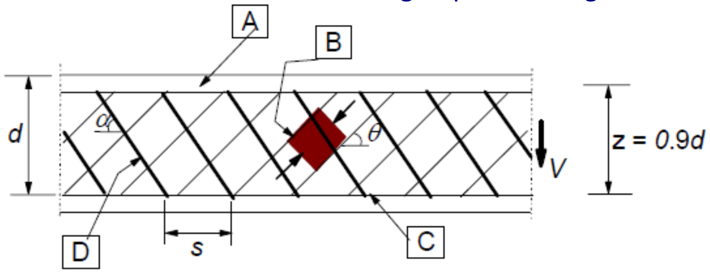
Shear solid slabs (rectangular cross-section)

– Truss model with no axial loading or prestressing –

Ref. EN 1992-1-1
Sec. 6.2.2 – Sec. 6.2.3
Cl. 9.2.1.2(3)
Sec. 9.2.2

UK N.A. to EC2

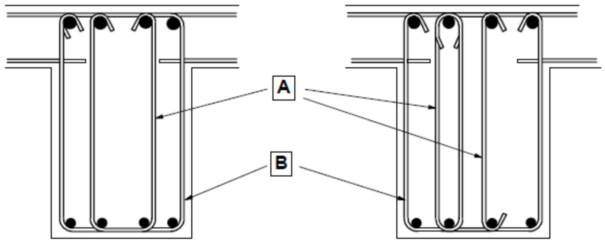


A - compression chord, B - struts, C - tensile chord, D - shear reinforcement

Figure 6.5

Geometry cross-section

Rectangular shape:
Width $b = b_w = 350$ mm
Height h = 600 mm
Effective depth d = 550 mm
Lever arm: z = 0,9d = 495 mm



A Inner link alternatives B Enclosing link

Figure 9.5

Analysis

Design shear occurring at cross-section being considered:
$V_{Ed} = 340$ kN.
Design shear stress @ cross-section:
$v_{Ed} = V_{Ed}/(b_w d) = 1,77$ MPa

Materials – characteristic/design strengths

Concrete C30/37

Strength class for concrete:

3.1.2(2)P - UK N.A. Table 3.1

$f_{ck} = 30\ N/mm^2 = 30$ MPa

3.1.2(4) - UK N.A.

$f_{cd} = \alpha_{cc} f_{ck}/\gamma_c = 0,85 f_{ck}/1,50 = 17,00\ N/mm^2$

Steel
Strength of shear reinforcement (Class B and C):

Table C.1 – Annex C

f_{ywk} = 500 N/mm^2 = 500 MPa $\leq$ 600 MPa (upper limit)

3.2.2(3)P - UK N.A.

$f_{ywd} = f_{ywk}/\gamma_s = f_{ywk}/1{,}15 = 0{,}87 f_{ywk}$ = 435 N/mm^2

Durability and cover to reinforcement
Nominal cover (for enclosing vertical links):
$c_{nom} = c_{min} + \Delta c_{dev}$ = 25 mm

Exp. (4.1)

Tensile reinforcement
Area of the tensile reinforcement, which extends ≥ (l_{bd} + d) beyond the section considered (see figure above):
A_{sl} = 600 mm^2

Shear resistance (without shear reinforcement)
Design value for the shear stress resistance:
$v_{Rd,C} = \max[C_{Rd,c}k(100\rho_l f_{ck})^{1/3};\ 0{,}035k^{3/2}f_{ck}^{1/2}]$ = 0,41 MPa
$C_{Rd,c}$ = 0,12; $k = 1 + (200/d)^{1/2}$ = 1,60 $\leq$ 2,0; $\rho_l = A_{sl}/(b_w d)$ = 0,003 $\leq$ 0,02
$V_{Ed} \leq 0{,}5 b_w d \nu f_{cd}$ = 558 kN (satisfactory).

Sec. 6.2.2
Exp. (6.2.a) – (6.2.b) – (6.3N).

Cl. 6.2.2(6) – Exp. (6.5)

Shear reinforcement required
v_{Ed} = 1,77 MPa $\geq$ $v_{Rd,C}$ = 0,41 MPa.

Cl. 6.2.1(3).

Detailing
Shear reinforcement for beam (stirrups and bent-up bars):
β_3 = 0,50 (chosen).
Design shear stress occurring at cross-section being considered v_{Ed} = 1,77 MPa (with: 0,88 MPa @stirrups and 0,88 MPa @bet-up bars).
Chosen cotθ = 1,00 with 1,00 $\leq$ cotθ $\leq$ 2,50.

Cl. 9.2.2(4) – Note

6.2.3(2) - UK N.A.
Cl. 6.2.3(2) – Note – Exp. (6.7N)

Stirrups
N = 1 enclosing links with diameter H10 mm, and no inner links.
Longitudinal spacing (vertical links): s = 190 mm (chosen).
α = 90° (angle between vertical links and the beam axis).

Bent-up bars
2H16 bars (A_{sw} = 402 mm^2) bent-up at an angle from the bottom of the beam α = 45° (for n = 2 times) with a longitudinal spacing:

$$s = \frac{z}{n}(\cot\theta + \cot\alpha) = 495 \text{ mm}$$

▶ Maximum longitudinal spacing (vertical links): — Cl. 9.2.2(6) - Exp.(9.6N)

$s_{l,max}$ = 0,75d(1 + cotα) = 413 mm.

Compression longitudinal reinforcement (diameter ϕ_{com}) which is included in the resistance calculation: H20 — Cl. 9.2.1.2(3)

$s_{l,max} = 15\phi_{com}$ = 300 mm

MIN[413; 300] = 300 mm ≥ s = 190 mm (satisfactory)

▶ Maximum transverse spacing of shear reinforcement (vertical links): — Cl. 9.2.2(8) – Exp. (9.8N)

$s_{t,max}$ = MIN[0,75d; 600 mm] = 413 mm

Transverse vertical spacing:

$$s_t = \frac{b_w - 2c_{nom} - \phi_{Enclink}}{N_{innerLink} + 1} = 290 \text{ mm} \le s_{t,max} \text{ (satisfactory)}$$

(where $N_{innerLink}$ = number of inner links).

▶ Minimum shear reinforcement (for sinα = 1): — Cl. 9.2.2(5) – Exp. (9.4) – (9.5N)

$$\left(\frac{A_{sw}}{s}\right)_{min} = \rho_{w,min} b_w \sin\alpha = \frac{0{,}08\sqrt{f_{ck}}}{f_{ywk}} b_w = 0{,}307 \text{ mm}^2/\text{mm} = 307 \text{ mm}^2/\text{m}.$$

A_{sw}/s = 0,827 mm²/mm = 827 mm²/m ≥ $(A_{sw}/s)_{min}$ (staisfactory).

Shear capacity (α* = MIN[$\alpha_{stirrups}$; $\alpha_{bentupbars}$])

ν_1 = 0,6·(1 − f_{ck}/250)·(1 − 0,5·cosα*) = 0,341. — 6.2.3(3) - UK N.A.

Capacity of the concrete section with vertical and inclined shear reinforcement to act as a strut:

$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd}/(\cot\theta + \tan\theta)$ = 502642 N (stirrups) — Exp. (6.9)

(cotθ + tanθ) = 2,00

$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd}(\cot\theta + \cot\alpha)/(1 + \cot^2\theta)$ = 1005284 N (inclined bars) — Exp. (6.14)

(cotθ + cotα) = 2,00

(1 + cot²θ) = 2,00

$V_{Rd,max}$ = MIN[502642; 1005284] = 502642 N (<u>used in calculations</u>).

Shear reinforcement (stirrups)

With: A_{sw}/s = 0,827 mm²/mm = 827 mm²/m

cotθ = 1,00

$V_{Rd,s} = (A_{sw}/s) z f_{ywd} \cot\theta$ = 177928 N — Exp. (6.8)

MIN[$V_{Rd,s}$; $V_{Rd,max}$]/(b_wd) = 0,92 MPa > v_{Ed} = 0,88 MPa (satisfactory).

Shear reinforcement (bent-up bars)

With: A_{sw}/s = 0,812 mm²/mm = 812 mm²/m

$(\cot\theta + \cot\alpha) = 2{,}00$
$\sin\alpha = 0{,}71$
$V_{Rd,s} = (A_{sw}/s)zf_{ywd}(\cot\theta + \cot\alpha)\sin\alpha = 247256$ N — **Exp. (6.13)**
$\text{MIN}[V_{Rd,s};\ V_{Rd,max}]/(b_w d) = 1{,}28$ MPa > $v_{Ed} = 0{,}88$ MPa (satisfactory).

Note

Provide H10 mm diameter enclosing links, (and no inner links) at 190 mm centres.
Provide 2H16 bent-up at α = 45° for n = 2 times with a longitudinal spacing s ≤ 495 mm.

Maximum effective cross-sectional area of the shear reinforcement (cotθ = 1) — **Cl. 6.2.3(3) – Note 4**

► Stirrups: — **Exp. (6.12)**

$$\left(\frac{A_{sw,max}}{s}\right) \le \frac{1}{2}\alpha_{cv}\nu_1\frac{f_{cd}}{f_{ywd}}b_w = 2{,}336\ \text{mm}^2/\text{mm} = 2336\ \text{mm}^2/\text{m}$$

► Bent-up bars: — **Cl. 6.2.3(3) – Note**, **Exp. (6.15)**

$$\left(\frac{A_{sw,max}}{s}\right) \le \frac{1}{2\sin\alpha}\alpha_{cv}\nu_1\frac{f_{cd}}{f_{ywd}}b_w = 3{,}303\ \text{mm}^2/\text{mm} = 3303\ \text{mm}^2/\text{m}.$$